

Bayesian Approaches to Clinical Trials and Health-Care Evaluation

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Bayesian Approaches to Clinical Trials and Health-Care Evaluation

David J. Spiegelhalter

MRC Biostatistics Unit, Cambridge, UK

Keith R. Abrams

University of Leicester, UK

Jonathan P. Myles

Cancer Research UK, London, UK



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Preface

This book began life as a review of Bayesian methods in health technology assessment commissioned by the UK National Health Service Research and Development Programme, which appeared as Spiegelhalter *et al.* (2000). It was then thought to be a good idea to change the review into a basic introduction to Bayesian methods which also tried to cover the field of clinical trials and health-care evaluation. We did not realise the amount of work this would involve.

We are very grateful to all those who have read all or part of the manuscript and given such generous comments, particularly David Jones, Laurence Freedman, Mahesh Parmar, Tony Ades, Julian Higgins, Nicola Cooper, Cosetta Minelli, Alex Sutton and Denise Kendrick. Unfortunately, by tradition, we must take full responsibility for all errors and idiosyncrasies. Our particular thanks go to Daniel Farewell for writing the BANDY program, and Nick Freemantle for providing data. The University of Leicester provided the second author with study leave, during which part of this work was carried out. Finally, we must thank Rob Calver and Siân Jones at Wiley for being so patient with the repeated excuses for delay: in the words of Douglas Adams (1952–2001), “I love deadlines. I especially like the whooshing sound they make as they go flying by”. We hope it has been worth the wait.

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